Constructive Interference in Linear Precoding Systems: Power Allocation and User Selection

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***Abstract*—The exploitation of interference in a constructive manner has recently been proposed for the downlink of multiuser, multi-antenna transmitters. This novel linear precoding technique, herein referred to as constructive interference zero forcing (CIZF) precoding, has exhibited substantial gains over conventional approaches; the concept is to cancel, on a symbol-by-symbol basis, only the interfering users that do not add to the intended signal power. In this paper, the power allocation problem towards maximizing the performance of a CIZF system with respect to some metric (throughput or fairness) is investigated. What is more, it is shown that the performance of the novel precoding scheme can be further boosted by choosing some of the constructive multiuser interference terms in the precoder design. Finally, motivated by the significant effect of user selection on conventional, zero forcing (ZF) precoding, the problem of user selection for the novel precoding method is tackled. A new iterative, low complexity algorithm for user selection in CIZF is developed. Simulation results are provided to display the gains of the algorithm compared to known user selection approaches.**

I. Introduction

The capacity of the multiple input multiple output (MIMO) broadcast channel (BC) can be reached by non-linear precoding methods, namely dirty paper coding (DPC) [1]. However, linear precoding methods, like zero forcing (ZF) precoding, can still attain the channel capacity in a multiuser environment [2]–[4], while proven more realistic in terms of practical implementation. Linear precoding techniques, especially ZF, have been extensively investigated in [3], [5] and the references therein. In these cases, ZF precoding constitutes a simple precoder design solution. By inverting the channel, multiuser interferences are cancelled and the precoding design problem is reduced to a power allocation problem over new equivalent channels; hence a simple concave optimization problem [6] needs to be solved. To maximize the throughput (sum-rate, SR), the well known water-filling solution can be straightforwardly applied [7]. To maximize the minimum offered rate (i.e. the fairness problem), the problem is still convex and thus solvable [5]. The key assumption of all the above considerations however is the assumption of Gaussian signaling.

The concept of constructive interference linear precoding, initially proposed in [8] for code division multiple access (CDMA) systems and then extended to apply for MIMO communications in [9], is based on the multiuser interference cancellation concept of channel inversion. An example of the concept is described in Fig. 1. The novelty of this precoder design lies in considering practical constellations and allowing users that add up to the intended user's signal power to interfere. This is referred to as constructive interference (CI) and it can be exploited by acknowledging each users' channel and modulated signal. The problem of power allocation in constructive interference zero forcing (CIZF) precoding techniques has not been studied in existing literature. Existing works on this topic only assumed CIZF precoding with equal power allocation for all users [9].

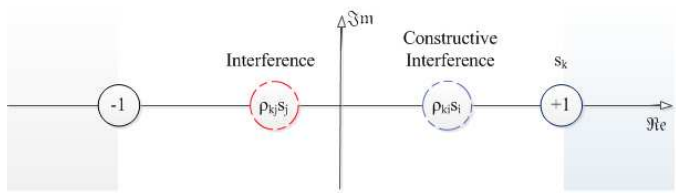

Fig. 1. The constructive interference (CI) concept over binary phase shift keying (BPSK) modulations: The k-th user's transmit symbol is $s_k = +1$. The i-th user's symbol, s_i, multiplied by the cross-correlation between the k-th and the j-th user's channels, ρ_{kj} (see Sec. II-B), is a vector that when added to s_k will move the resulting vector further away from the decision threshold (0 for BPSK). Subsequently, not cancelling this user will benefit the k-th user. On the other hand, the j-th user is still interfering thus needs to be cancelled by the precoding design.

Another very important aspect of linear precoding is the user selection problem investigated in [3], [6]. ZF performance is increased when user channels are orthogonal to each other. Under the assumption of large random user sets, the probability of orthogonal users increases and with that the complexity of the user selection problem. Nevertheless, simple suboptimal algorithms in the existing literature provide substantial gains with affordable complexity. Based on existing methods, *Yoo et al* [2] proposed a low complexity, iterative user selection algorithm that allows ZF to achieve the performance of non-linear precoding [10] as the number of available for selection users grows to infinity.

The contribution of the present paper is twofold. Firstly, the effect of power allocation on CIZF precoding transmitters is investigated; a design parameter that has not been examined in existing literature. Secondly, motivated by the fact that user

selection can optimize the ZF performance, the problem of user selection in CIZF systems is defined and solved by a low complexity algorithm. This algorithm achieves substantial gains and approaches the performance of the optimal user selection, as derived by full space search.

The rest of the present paper is structured as follows. The considered system model is described with detail in Section II, where the concepts of conventional ZF and novel CIZF are also described. Section III explores the effects of power allocation in CI based linear precoding methods with the support of simulation results. In Section IV, the user selection problem is defined and solved via a novel heuristic algorithm. Conclusions are drawn in Section V.

Notation: Throughout the paper, $(\cdot)^\dagger$, $\mathfrak{Re}(\cdot)$ and $||\cdot||$, denote the conjugate transpose, the real part of complex elements and the Euclidean norm operations, respectively, while $[\cdot]_{ij}$ denotes the i,j-th element of a matrix. The element-wise matrix product is denoted by $\circ$. Bold face lower case characters denote column vectors and upper case denote matrices while the operator diag($\mathbf{x}$) produces a diagonal square matrix composed of the elements of $\mathbf{x}$. An identity matrix of size n is denoted by $\mathbf{I}_n$. Upper case calligraphic characters denote sets. The operation $\mathcal{A}-\mathcal{B}$ is the relative complement of $\mathcal{B}$ in $\mathcal{A}$, while $|\mathcal{A}|$, denotes the cardinality of a set.

II. System Model

A multi-user (MU) multiple input single output (MISO) network consisting of one transmitter with N_t antennas and $K \geq N_t$ single antenna receivers, is considered. At each time, the transmitter serves exactly $K = N_t$ users which are selected either randomly or based on a selection scheme as described in Sec. IV. The received signal at the k-th user can be expressed as

$$y_k = \mathbf{h}_k^\dagger \mathbf{x} + n_k, \tag{1}$$

where $\mathbf{h}_k$ is an $N_t \times 1$ vector composed of the channel coefficients between the k-th user and the N_t antennas of the source, $\mathbf{x}$ is an $N_t \times 1$ vector of transmitted symbols and n_k is the independent identically distributed (i.i.d) zero mean Additive White Gaussian Noise (AWGN) measured at the k-th user's receive antenna. The noise is assumed normalized, thus $\mathcal{E}\left\{|n_k|^2\right\} = 1$. In matrix form, this MU MISO BC reads as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2}$$

where $\mathbf{y} = [y_1, y_2, \ldots, y_{N_t}]^\dagger$, $\mathbf{x} = [x_1, x_2, \ldots, x_{N_t}]^\dagger$, $\mathbf{H}$ is the $N_t \times N_t$ square matrix that contains the user complex vector channels, i.e. $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2 \ldots, \mathbf{h}_{N_t}]^\dagger$ and $\mathbf{n} = [n_1, n_2, \ldots, n_{N_t}]^\dagger$. The transmitter linearly precodes the information symbols:

$$\mathbf{x} = \mathbf{W}\mathbf{P}^{1/2}\mathbf{s}, \tag{3}$$

where the $N_t \times N_t$ matrix $\mathbf{W}$ is the precoding matrix, s_k denotes the symbol for the k-th destination and $\mathbf{s} = [s_1,\ s_2, \ldots, s_K]^\dagger$ with $\mathbb{E}[\mathbf{s}\mathbf{s}^\dagger] = \mathbf{I}$ and $\mathbf{P}^{1/2} = \mathrm{diag}([\sqrt{p_1}, \sqrt{p_2} \ldots \sqrt{p_k}])$ is a diagonal $K \times K$ matrix composed of the transmit powers allocated to the k users[1]. For shortness and since the results can be straightforwardly generalized for higher order constellations, in this study only real valued signals will be assumed (binary phase shift keying-BPSK modulation), hence

$$s_i = \pm 1,\ i = 1 \ldots N_t. \tag{4}$$

A. Zero Forcing beamforming

Transmit beamforming is a multiuser precoding technique that separates user data streams in different parallel beamforming directions [3]. A linear precoding technique with reasonable computational complexity that still achieves full spatial multiplexing and multiuser diversity gains, is ZF precoding [3], [11], [12]. The ability of ZF to fully cancel out multiuser interference makes it useful in the high Signal to Noise Ratio (SNR) regime. However, it performs far from the optimal in the noise limited regime. In addition, it can only simultaneously serve as many single antenna users as the number of transmit antennas. A common solution for the ZF precoding matrix is the pseudo-inverse of the $K \times N_t$ channel matrix. Under a total power constraint, the pseudo-inverse is the optimal solution (rather than any generalized inverse) in terms of maximum SR and maximum fairness [5]. The precoding matrix can be expressed as

$$\mathbf{W} = \mathbf{H}^\dagger \mathbf{R}^{-1} \mathbf{T}, \tag{5}$$

where we define the matrix $\mathbf{R}$ as

$$\mathbf{R} = \mathbf{H}\mathbf{H}^\dagger. \tag{6}$$

The matrix $\mathbf{T}$ has been introduced by [9] to model the CI scheme as explained in the next subsection. The general model described by (5), for

$$\mathbf{T} = \mathbf{I} \circ \mathbf{R}, \tag{7}$$

where the non zero elements of $\mathbf{T}$ are $\tau_{kk} = \sum \mathbf{h}_k^\dagger \mathbf{h}_k$, will yield the conventional ZF design. The complete cancellation of interferences in this case, will reduce the k-th users received signal to

$$y_k = \mathbf{h}_k^\dagger \sqrt{p_k} \mathbf{w}_k s_k + n_k, \tag{8}$$

where $\mathbf{w}_k$ is the k-th column of the total precoding matrix. Assuming uniform power allocation across users, as in [9], the total power constraint over the transmit antennas P_{tot} will yield [5]:

$$\mathcal{E}\{||\mathbf{x}||^2\} = \mathrm{Tr}\{\mathbf{x}\mathbf{x}^\dagger\} \leq P_{tot}. \tag{9}$$

From (3) and (9), the transmit power allocated to the k-th user becomes

$$\sqrt{p}_k = \sqrt{\frac{P_{tot}}{\mathrm{Tr}\{\mathbf{T}^\dagger \mathbf{R}^{-1} \mathbf{T}\}}},\ \forall\ k. \tag{10}$$

[1]The notion of transmit power allocated to a user is explained in [3].

By examining (10), it is clear that the transmit power allocated to the k-th user is a function of the precoder design. In precoding, channel inversion i.e. projecting the actual channels on orthogonal dimensions, leads to the reduction of each users effective channel. Subsequently, since the precoders are not normalized, the sum of the individual powers allocated to each user ($\sqrt{p_k}$) is not equal to the sum of powers transmitted (9). The notion of individual user consumption can be introduced to better explain this power loss due to the precoding. Finally, the k-th user SINR for the ZF precoding will read as

$$\mathrm{SINR}_k^{\mathrm{ZF}} = |\tau_{kk}|^2 p_k. \tag{11}$$

B. Constructive Interference Zero Forcing beamforming

The CIZF scheme, introduced in [9], allows the so-called constructive multiuser interference (cross-interference) to be added to the useful signal at each receiver. In general, given the full channel state information available at the transmitter and acknowledging the signal constellation, the CIZF scheme does not suppress the part of the cross-interference that is constructive and thus increases the power of the useful signal. A simple example of this concept is explained in Fig. 1.

As discussed with detail in [9], the symbol to symbol multiuser interference results from the i,j-th element of the matrix $\mathbf{R}$: $\rho_{ij} = \sum_{n=1}^{N_t} h_{in} \cdot ({h_{jn}}^\dagger)$. In the CI scenario, the received signal of (8) will become

$$y_k = \tau_{kk}\sqrt{p_k}s_k + \sum_{j\neq k}\mathrm{CI}_{kj} + n_k \tag{12}$$

where $\mathrm{CI}_{kj} = \tau_{ki}\sqrt{p_j}s_j$ denotes the constructive cross-interference from the j-th data flow (j-th user) to the k-th user. Subsequently, the k-th user's signal to interference plus noise ratio (SINR) will read as

$$\mathrm{SINR}_k = \sum_{j=1}^{K} |\tau_{kj}|^2 p_j. \tag{13}$$

Let us define as $\mathbf{G} = \mathrm{diag}(\mathbf{s}) \cdot \mathfrak{Re}(\mathbf{R}) \cdot \mathrm{diag}(\mathbf{s})$, which yields

$$\mathbf{G} = \begin{pmatrix} s_1\mathfrak{Re}(\rho_{11})s_1 & \dots & \\ \vdots & \ddots & \\ & & s_k\mathfrak{Re}(\rho_{kj})s_j \\ & & \end{pmatrix}. \tag{14}$$

In order to indicate the cross-interference as CI, the signal constellation needs to be accounted for. Subsequently, the terms that position the received signal into the decision region of the transmitted symbols are beneficial and thus not cancelled by the precoding design. For the simple case of BPSK modulation, the cross-interference generated by the j-th data flow to the k-th destination, is considered to be constructive when

$$s_k\mathfrak{Re}(\rho_{kj})s_j > 0, \tag{15}$$

which can be expressed as $\mathbf{G}_{kj} > 0$. Thus the CIZF precoder is deduced from

$$\tau_{kk} = \rho_{kk} \tag{16}$$
$$\tau_{ki} = \begin{cases} \rho_{ki}, & \text{If } [\mathbf{G}]_{kj} > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Therefore, the precoding matrix is computed on a symbol-by-symbol basis.

III. Power Allocation in Linear Precoding

The impact of power allocation (PA) on the CIZF has not been addressed in existing literature on this topic [8], [9]. Therein, the problem was simplified by a uniform power allocation assumption, as defined in (10). In general, PA is performed to the end of maximizing some performance metric. The performance metrics commonly addressed in literature involve either the total throughput performance (i.e. max SR criterion) or the SINR level of the worst user (i.e. max fairness criterion). Another important parameter in linear precoding is the type of constraints that will be assumed. Usually, a total sum power constraint simplifies the analysis and provides better results since the available power is freely allocated across antennas. Herein, two objective functions of the achievable user rates that ensure maximum fairness (availability) and maximum SR (throughput), are considered. More specifically, the optimization problem reads as

$$\max_{\mathbf{P}\geq\mathbf{0}} f(\mathbf{P}) \tag{17}$$
$$\text{s.t.} \sum_{i=1}^{N_t}\sum_{j=1}^{K} |\omega_{ij}|^2 p_j \leq P_{tot}$$

where ω_{ij} is the i,j-th element of $\mathbf{W}$ and the objective function f is given by [5]:

$$f(\mathbf{P}) = \begin{cases} \sum_k \log_2(1+\mathrm{SINR}_k), & \text{Throughput} \\ \min_k \mathrm{SINR}_k, & \text{Fairness} \end{cases} \tag{18}$$

where SINR_k is given by (13) and $1 \leq k \leq N_t$. Based on (18), the objective function is concave in $\mathbf{P} \geq 0$, for both scenarios and therefore the optimization problem is a simple concave maximization with one linear constraint. It is worth noting that for the case of the maximum throughput, the optimization problem can be solved using the water-filling solution. The problem of allocating the power to the end of maximising some system performance metric is discussed in the following Section (III-A).

A. Power Allocation

An appropriate PA scheme distributes the total available power to the data flows in a way that maximizes an objective function of the achievable rates. In the case of conventional ZF precoding, the max throughput PA problem, under a total power constraint, reads as in (17) with objective function

$$f(\mathbf{P}) = \sum_k \log_2\left(1+\mathrm{SINR}_k^{\mathrm{ZF}}\right), \tag{19}$$

where the $\mathrm{SINR}_k^{\mathrm{ZF}}$ is given by (11).

In order to be able to easily show the impact of PA on CIZF and maintain concavity for the formulated optimization problems, we assume that the equivalent CIZF channel (with the modulation-based CI) refers to Gaussian inputs; this assumption allows to approximate the channel capacity of the system with the simple $\log$-based Shannon expressions. It should be clarified here, that the optimal power allocation problem for linear precoders under the constraint of finite input alphabets is a highly complex problem. The most recent attempt to solve it can be found in [13] where a heuristic optimization algorithm is developed. However, in the present paper, a preliminary study to exhibit the impact of PA on CIZF is performed, hence the strictly optimal solution is beyond the scope of the present work.

1) Simulation Results: The effect of power allocation in the CIZF precoding design is plotted in Fig. 2. The power allocation problem (18) has been solved using the CVX tool in MATLAB [14]. Simulations where carried for 100 channel instances, and for $K = N_t = 4$. In Fig. 2, the gain from CIZF with uniform power allocation, as proposed in [9], over the conventional ZF is evident by comparing the dashed lines. The novel result, depicted in Fig. 2, is that power allocation further boosts the CIZF gain (continuous lines). More specifically, for the conventional ZF scheme power allocation introduces approximately 1 dB of gain over the uniform allocation. However, when PA is applied in CIZF, then more than 2 dB gain can be gleaned. It is therefore concluded that power allocation over the CIZF precoding scheme is an important aspect that introduces significant gains. Finally, in the same figure, the realistic region where the results apply is defined by a dashed line. This restriction comes from the acknowledgement of BPSK modulation as a practical constellation choice. The alleviation of this restriction via adaptive modulation methods is part of the future extensions of this work.

B. Power Constrained Transmission

In the present section, the existence of redundant CI interference terms is discussed. As can be seen from (10) and (13), the consideration of non-zero, off diagonal elements in the precoding matrices, i.e. CI terms, has a double impact on each user's SINR and thus the sum system capacity; from one hand, it increases the expression $\sum_i |\tau_{ki}|^2 p_k$ by adding more positive terms in the summation, but on the other hand, it changes the power allocated to each user in (9), (10). Therefore, a CI term in the CIZF is not always beneficial for the system performance. This partially constructive interference zero forcing (P-CIZF) scheme examines the tradeoff between the positive and the negative impact of an CI term by searching all combinations and selecting the most beneficial set of CI terms. The P-CIZF scheme can be formulated as

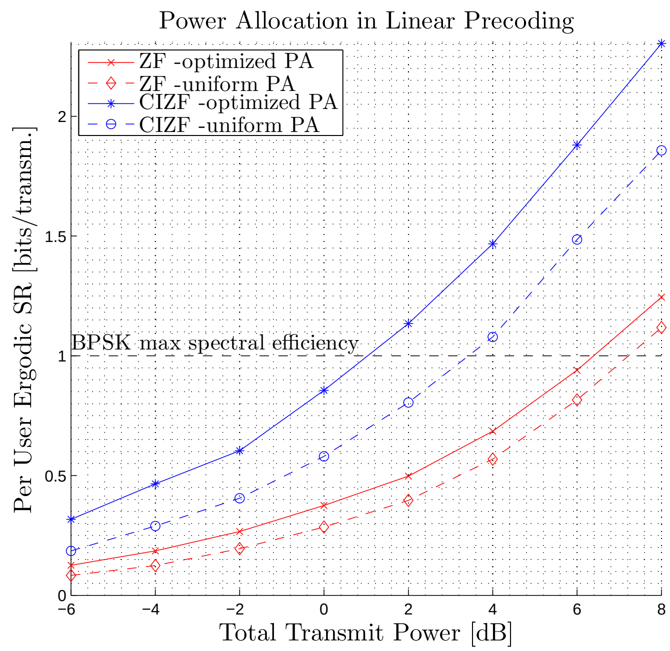

Fig. 2. Per user spectral efficiency of conventional ZF precoding under uniform and optimized PA, compared to the efficiency of the novel CIZF precoding under equivalent PA assumptions. PA optimization for max system throughput. The modulation constrained threshold for BPSK is also plotted.

$$\max_{\{\mathbf{p},\mathbf{T}^{(\mathcal{S})}\}} f(\mathbf{p},\mathbf{T}^{(s)}) \tag{20}$$

$$\text{s.t.} \quad \mathbf{T}^{(\mathcal{S})} \subseteq \mathbf{T}^{(\mathcal{S}_{tot})},$$

$$\sum_{i=1}^{M}\sum_{j=1}^{K} |\omega_{ij}|^2 p_j \le P_{tot}. \tag{21}$$

where

$$\mathcal{S} \subseteq \mathcal{S}_{tot} = 2^m,$$

with $m = |[\mathbf{G}]_{ki} > 0|$, the number of CI terms. The above definition means that the Partially-CIZF scheme searches all the possible combinations (2^m) of the CI terms and holds the one that maximizes the objective function considered. The optimization problem (20), will be solved under uniform (10) and optimal (21) power allocation considerations. Intuitively, the second consideration provides more flexibility in the design but finding the strictly beneficial terms while at the same time optimizing the PA is a highly complex procedure; for every possible combination of the CI terms, a new convex optimization PA problem is solved.

As the purpose of this work is to demonstrate this interesting trade-off, more practical implementation of the P-CIZF scheme are beyond the scope of this paper.

1) Simulation Results: In Fig. 3 the improvements of P-CIZF over the CIZF scheme are depicted. The performance is evaluated under uniform and optimized power allocation. Starting with the CIZF scheme under uniform power allocation, in Fig. 3, finding the strictly beneficial CI terms provides

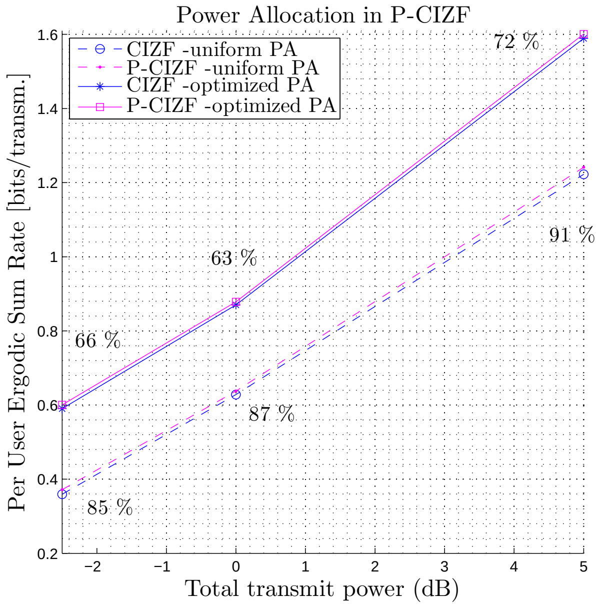

Fig. 3. Evaluation of the P-CIZF compared to fully CIZF under equal and optimal power allocation. The percentages of the constructive terms maintained in the P-CIZF scheme, for each SNR point are also presented.

some gains. In each point of the figure, the percentage of the CI terms kept in the P-CIZF scheme over the total CI terms of the CIZF scheme is also depicted. Focusing on the dotted curves, in the uniform power allocation case, it is apparent that maintaining approximately 88% of the total number of CI terms, provides small gains. These results exhibit the optimality of this approach, while analytical proof of the optimality is part of future work. The continuous curves in Fig. 3 correspond to an optimal (with respect to maximizing the total SR) power allocation assumption in P-CIZF precoding and again some gains are gleaned; thus the strict optimality of this approach is exhibited. Since the degrees of freedom in the precoder design are increased, less CI terms are maintained (approximately 65%).

Intuitively, the above results can be explained by the fact that allowing only strictly beneficial terms in the precoder reduces power consumption, thus allowing for more power to be allocated to the users without exceeding the total power constraint imposed on the transmit antennas. It should be noted here, however, that the effects of this approach on the minimum supported rate (fairness) are not examined in the present work. Finally, higher gains of this approach could be gleaned over larger user sets, but by exponentially increasing the iterations of the searching algorithm. The investigation of such scenarios is part of future work.

IV. User selection

In this section, the user selection problem is formulated and a novel selection technique based on the exploitation of CI is proposed. Considering the decoupled nature between the user selection and the PA problem, as proven in [3], the performance of user selection is not affected by PA. However, in accordance with the previous results and more importantly to maximise the overall performance of the assumed system, PA is optimized independently after the user selection process.

A. User Selection Methods

1) Orthogonal user selection: ZF beamforming has the potential of approaching the optimal channel capacity, otherwise only achievable by DPC, when all the users are perfectly orthogonal to each other, as proven by [3]. The same authors provided a heuristic low complexity user selection algorithm, namely the *semi-orthogonal user selection* (SUS) algorithm, which was proven to select the optimal users, as the number of available users approaches infinity. However, it has never been applied in the CIZF framework. Due to lack of space, the reader is referred to [2], [3] for more details on this algorithm. It should be mentioned here, that the orthogonal user selection does not take into account the CI during the selection procedure. However, to maintain fairness in the study, after the selection, any CI terms that exist are maintained.

2) Optimal Constructive Interference User Selection: The investigated precoding scheme strongly depends on the transmitted signal (user constellation) and therefore the above conventional user selection schemes become unsuitable. A CIZF-based user selection metric should also consider the transmitted symbols, since the precoding matrix is defined via the CI and affects the final performance. In order to attain an upper bound for any CIZF-based user selection policy, all the possible combinations of users $\mathcal{Q}$ are examined and out of them, the combination $\mathcal{U}$ that maximizes the system throughput is chosen:

$$\mathcal{U} = \arg_{\mathcal{U}} \max_{\mathcal{U}\in\mathcal{Q}} \sum_{m} \log_2\left(1+\mathrm{SINR}_m\right), \tag{22}$$

where SINR_m is given by (13) under a uniform power allocation assumption, i.e. (10). It should be stressed that this method relies on exhaustive search over all possible combinations of users. As a result, a searching algorithm requires $\binom{K}{N_t} = K!/(K-N_t)!$ iterations in order to decide about the optimal combination at each transmission. Considering that user selection methods perform better as the number of users increases [3], i.e. as $K >> N_t$, then the optimal solution becomes difficult to compute. In the scope of this work, a simpler heuristic algorithm is presented hereafter.

3) Semi parallel user selection: Inspired by the concept of user orthogonality, the purpose of this selection method is dual: users with CI need to be selected and furthermore these users need to be aligned (rather than orthogonal) so that the aggregate beneficial receive power is increased. Following this concept the *semi-parallel user selection* (SPUS) algorithm, provided in pseudo-code in Alg. 1, has been developed. An analytic description of the algorithm follows.

Initially, the algorithm accepts as input the CI matrix $\mathbf{G}$. The first step is to choose the user with the larger diagonal element. For this user, the cross-correlation elements with all other users are stored in the buffer vector $\mathbf{c}_{(i)}$ where i is the iteration

counter. Also the sets $\mathcal{S}, \mathcal{T}$ that include the available and the selected users, are initialized and updated in every iteration. Cross-correlation is the inner product of the vector channels of two users and represents the level of orthogonality between the users. In this scenario, the purpose is to have as little orthogonality as possible so as to increase the received CI. In each of the M iterations, (Step 2) the user with the strongest element in $\mathbf{c}_{(i)}$ is chosen. Then this user's corresponding cross-correlations with all the users is added in the buffer matrix. By adding the cross-correlation of each selected user in the buffer matrix, a metric for the subspace of the previously selected users is created, since the aim of the algorithm is to find the most parallel users to the subspace spanned by the ones already selected. This is achieved by choosing the strongest element of the $\mathbf{c}$ vector in each iteration. Since in the previous steps no guarantee exists that a selected user has only CI towards the selected ones, in Step 3, the residual non-CI terms are removed from the precoding matrix. The developed heuristic, iterative algorithm runs for exactly K iterations.

Simulation Results: A comparison in terms of maximum SR performance of the algorithms described in the previous section is presented in Fig. 4, where the performance of these algorithms was studied with optimized PA to maximize the total throughput. In this figure, the gain of the optimal user selection algorithm compared to existing approaches is clear. The best algorithm for ZF precoding, i.e. SUS [3], performs better than assuming no selection; however, approximately 6 dB loss is expected over the optimal CIZF selection. This observation emanates the need for better user selection algorithms, when exploiting the benefits of CI. Accounting also for the complexity of an exhaustive search selection algorithm, as discussed in the previous section, a less complex heuristic algorithm is further necessitated. In this direction, SPUS has been developed. In Fig. 4, the close to optimal performance of the developed algorithm is clear; the proposed technique performs less than 1 dB away from the optimal selection.

Furthermore, the performance of the developed algorithm under fairness maximization PA is examined in Fig. 5. Results indicate, that when the SPUS algorithm is combined with max fairness PA optimization, the performance degradation with respect to the optimal selection policy, is relatively small. Therefore, the proposed algorithm, developed for maximising the total throughput of the system, does not severely compromise the fairness of the system. By comparing Fig. 4 and 5, it is also noted that the the minimum user rates are not far from the average rate. This result indicates that the variance between the user rates is kept in reasonable levels when user selection is combined with PA to optimize the total SR. It is therefore concluded that fairness is not severely compromised in user selection scenarios, even when PA optimization is performed to maximize to total system SR.

In Fig. 6 the performance of the discussed algorithms is studied with respect the size of the available user set. The beneficial effect of the increasing number of users is clear for all algorithms. What is more, for relatively small user pools the performance of the algorithms is beginning to saturate thus indicating that the main gains are gleaned for finite numbers of users, that are in line with the dimensions of practical operating multiuser systems.

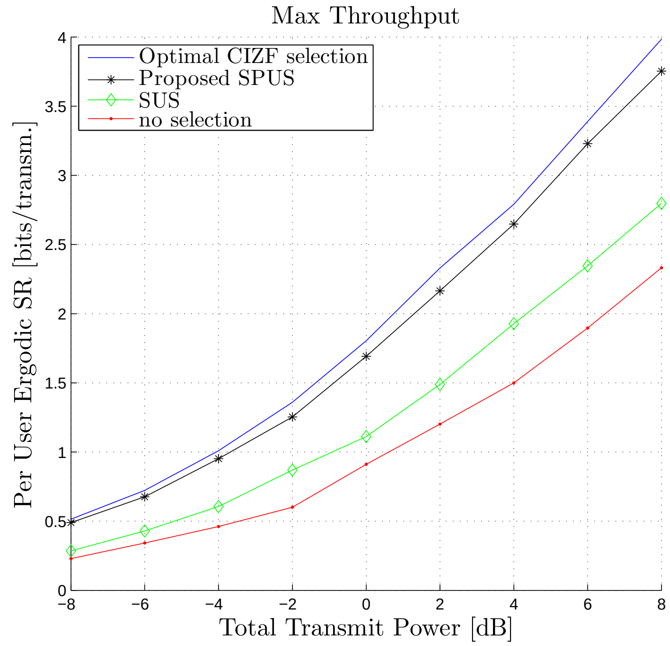

Fig. 4. Performance of user selection algorithms with respect to the total available transmit power. Results for $N_t = 4$ users selected out of a total pool of $K = 12$ users. PA has been optimized to maximise the total system throughput.

V. Conclusions and Future Work

The concept of constructive interference in linear precoding systems has been examined under the framework of power optimization for the maximization of the system performance. Results indicate that an excess of 2dB gain can be gleaned by optimizing the power allocation with the aim of increasing the total system throughput in constructive interference precoding systems. Moreover, as it has been shown, the individual user power consumption of these schemes can be further reduced, thus leading to some gains. Finally, the user selection problem has been tackled for the novel type of precoding and a heuristic, low complexity, iterative algorithm with close to optimal performance has been proposed.

Future extensions of this work include the investigation of constructive interference amongst users with different constellations in an adaptive modulation environment where the limitations induced by these practical constellations are alleviated.

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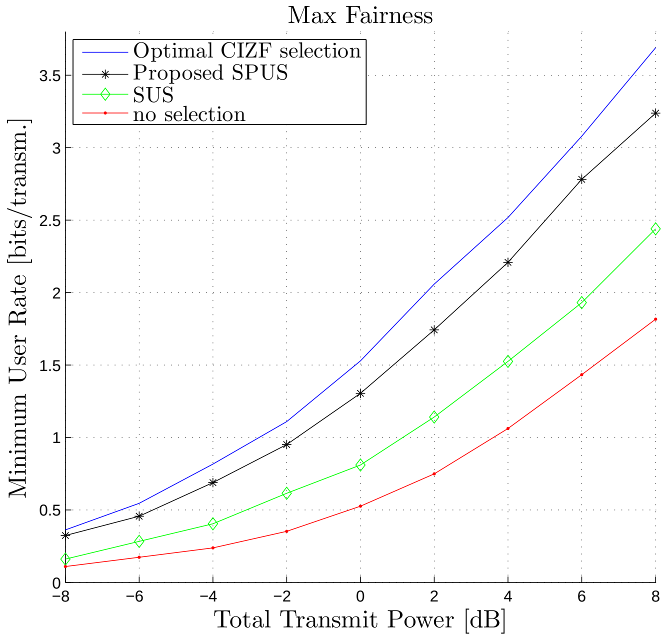

Fig. 5. Performance of user selection algorithms with respect to the total available transmit power. Results for $N_t = 4$ users selected out of a total pool of $K = 12$ users. PA has been optimized to maximise minimum user rate.

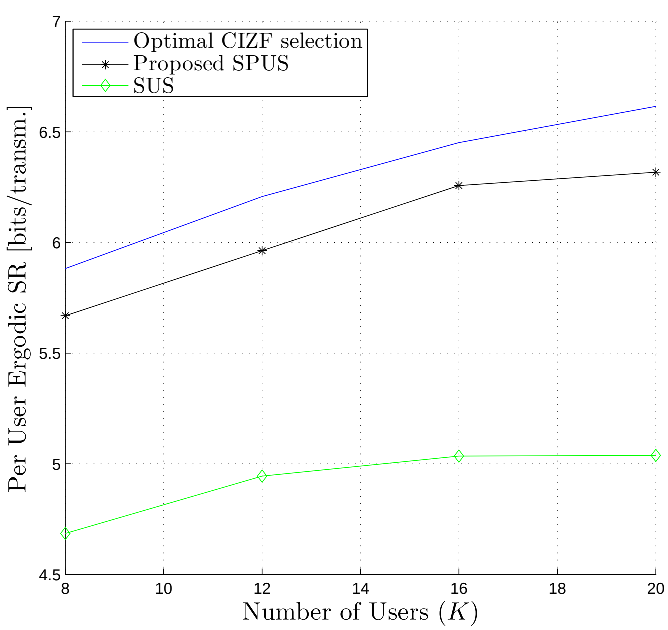

Fig. 6. Performance of user selection algorithms with respect to the total available transmit power. Results for $N_t = 4$ users selected out of a variable pool of users, for a fixed total transmit power of 15 dB.

Semi-Parallel User Selection (SPUS) algorithm

Output: $\mathbf{G}_{out}$

Input: $\mathbf{G} = \text{diag}(\mathbf{s}) \cdot \Re\mathfrak{e}(R) \cdot \text{diag}(\mathbf{s})$,

Step 1: Initialization

$\pi_{(0)} = \arg\max ||\mathbf{h}_k|| = \arg\max[\mathbf{G}]_{kk}$,

$\forall k = 1, ...M : \mathbf{c}_{(0)} = \mathbf{G}(\pi_{(0)}, k)$, $\mathcal{S}_{(0)} = \pi_{(0)}$

$\mathcal{T}_{(0)} = \{1, \ldots K\} - \{\pi_{(0)}\}$ set of unprocessed users.

for $i = 1 \to M$ **do**

  Step 2: Selection

  $\pi_{(i)} = \arg\max \mathbf{c}_{(i-1)}$, Provided that $\pi_{(i)} \in \mathcal{T}_{(i-1)}$;

  $\forall k = 1, ...M :$ $\mathbf{c}_{(i)} = \mathbf{G}(\pi_{(i-1)}, k) + \mathbf{G}(\pi_{(i)}, k)$;

  $\mathcal{T}_{(i)} = \mathcal{T}_{(i-1)} - \{\pi_{(i)}\}$;

  $\mathcal{S}_{(i)} = \mathcal{S}_{(i-1)} + \{\pi_{(i)}\}$;

end

Step 3: Output

$\mathbf{G}_{out} = \mathbf{G}(\mathcal{S}_{(M)})$;

for $m = 1 \to M$ **do**

  for $l = 1 \to M$ **do**

    if $\mathbf{G}(m, k) < 0$ **then**

      $\mathbf{G}_{out}(m, k) = 0$

    end

  end

end

Algorithm 1: Semi-Parallel User Selection Algorithm (SPUS)

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